

The Analysis of a Dwell Mechanism for Alpha -Stirling Engine

N.M. Dehelean and V. Ciupe

Abstract All engineers who work to improve the work efficiency of the α Stirling engine try to replace the crank mechanism of compression cylinder by another one. It could be useful a mechanism able to perform a dwell during the whole period of expansion. This stop of the compression piston has to enlarge the active area of the PV diagram. As consequence it is expected an efficiency improvement. This request could be very well accomplish by a cam mechanism. But this one is too large, too complicate, expansive and unreliable for solar systems as shown in [1]. Hence, the paper tries to find out a linkage able to perform an equivalent dwell as a cam mechanism. The purpose of the analysis is to find out the movement functions $y_C(t)$ and $y_E(t)$ to reveal if there are in a proper phase relation between them or not. The proper phase relation demonstrates that the mechanism is good for the application. Second, it must show that $y_E(t)$ has a large enough dwell, in accordance with the expansion stroke time of the expansion cylinder.

Keywords Bar mechanism · Crank rod mechanism · Dwell mechanism · Movement law · Stirling engine

1 On the Working Cycle of Stirling Engine

The PV diagram of α -Stirling engine it is far away from an ideal PV diagram. The principle schema of a α -Stirling engine is shown in Fig. 1. As seen the engine has two crank rod mechanisms, one for the compression cylinder and the other for the expansion cylinder. The movement functions for both pistons are sinusoidal, as shown in Fig. 2.

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Fig. 1 The standard α -Stirling engine

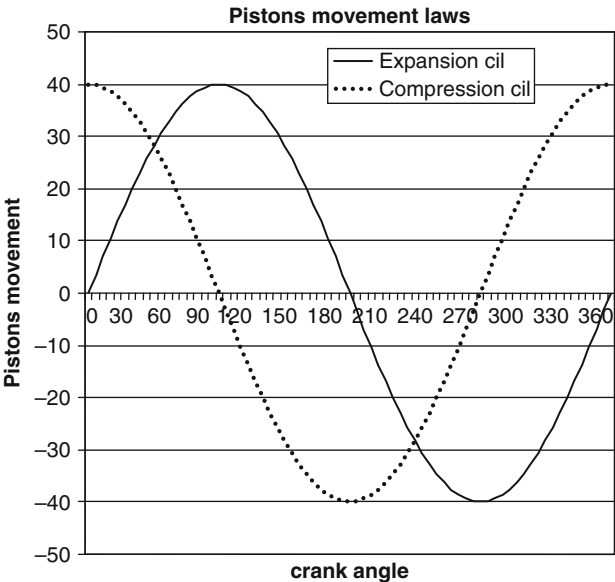
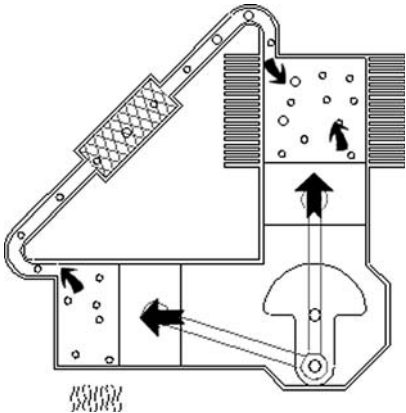


Fig. 2 Piston movement functions

Therefore, all the engineers who struggle to improve the work efficiency of the engine try to replace the crank mechanism of the compression cylinder by another one. It could be useful a mechanism able to perform a stop during the whole period of expansion. This stop of the compression piston has the role to enlarge the active area of the PV diagram shown in [2] and as a consequence it is expected an efficiency improvement in [4]. This request could be very well accomplish by a cam mechanism. But this one is too large, too complicate, expensive and unreliable. Hence, we try to find out a bar mechanism able to perform an equivalent stop as a cam mechanism.

In order to reach the dwell displacement function, without using a cam mechanism, the authors try to create a new bar mechanism.

2 A New Dwell Mechanism Dedicated to a-Stirling Engine

In [3] an acceptable dwell mechanism is expressed, able to be used in a α -Stirling engine. But this one is not good enough regarding the phases of the two pistons (Fig. 3). In order to improve this situation it is necessary to change the position of the articulation B between bars 2 and 3. It is preserved the relative position of the cylinder axis parallels. Hence the mechanism will become like the schema presented in Fig. 4.

The same reference xoy axes are placed in accordance with the Schmidt Analysis (Gustav Schmidt has made the classical Stirling Analysis in 1871).

Using the same reference we can replace the pseudo-sin function of compression cylinder, in Schmidt analysis by the new space function, in order to develop a new isothermal analysis of the engine.

3 On the Movement Functions of the Mechanism

The purpose of the analysis is to find out the movement functions $y_C(t)$ and $y_E(t)$ to reveal if there is a proper phase relation between them or not. The proper phase relation demonstrates that the mechanism is good for the application. Second, it must show that $y_E(t)$ has a large enough stop level, in accordance with the expansion stroke time of the expansion cylinder.

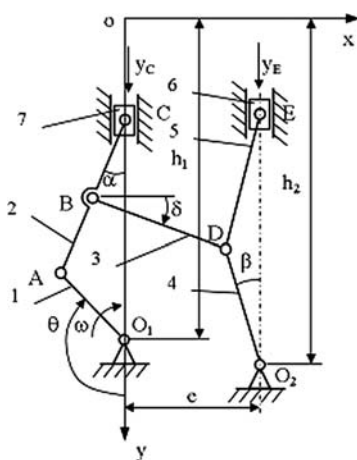


Fig. 3 General bar mechanism

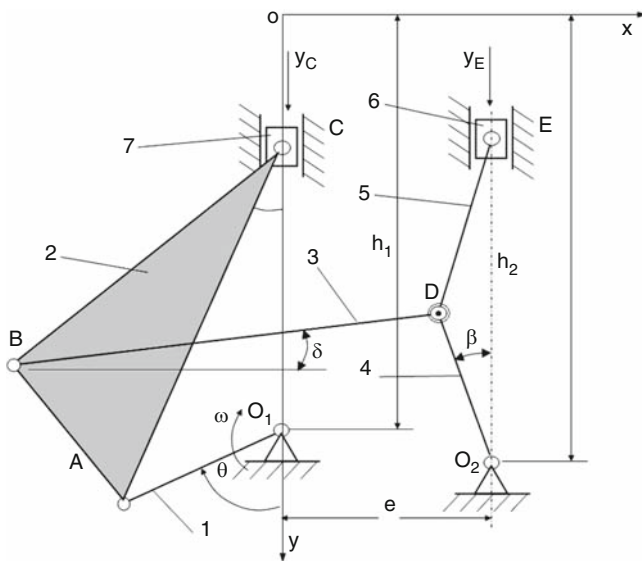


Fig. 4 The modified bar mechanism

The new displacement function $y_E(t)$ will be inserted in Schmidt analysis, instead of $\sin$ relation.

Notations used:

$O_1A = R_c$ rod

$AC = b$; $BB' = h$; $AB' = b_1$

$O_2D = R_2$

$BD = K$

The point $A(x_A, y_A)$ has a position expressed by the equation system (1). The angle $\theta = \theta(t)$ and $\theta_0 = \theta_0$ when the rod O_1A is oriented to oy axis.

$$\begin{cases} x_A = -R_{c\ rod} \sin(\theta) \\ y_A = h_1 + R_{c\ rod} \cos(\theta) \end{cases} \quad (1)$$

Using the $\sin$ theorem equation (2) can be written as follows:

$$\frac{b}{\sin(\pi - \theta)} = \frac{R_{c\ rod}}{\sin(\alpha)} \quad (2)$$

resulting the angle $\alpha = \alpha(t)$:

$$\alpha = \arcsin\left(\frac{R_{c\ rod}}{b} \sin \theta\right) \quad (3)$$

The point B(x_b , y_b) has a position expressed by the equation system:

$$\begin{cases} x_b = (b_1 - b) \sin \alpha - h \cos \alpha \\ y_b = h_1 + R_{c \text{ rod}} \cos(\theta) - b_1 \cos \alpha - h \sin \alpha \end{cases} \quad (4)$$

where b_1 , b , h are the geometrical parameters of the crank (Fig. 5).

The point C(x_C , y_C) has a position expressed by the equation system:

$$\begin{cases} x_c = 0 \\ y_c = h_1 + R_{c \text{ rod}} \cos(\theta) - b \cos \alpha \end{cases} \quad (5)$$

The point D(x_D , y_D) has a position expressed by the equation system:

$$\begin{cases} x_D = e - R_2 \sin \beta \\ y_D = h_2 + R_2 \cos \beta \end{cases} \quad (6)$$

The point E(x_E , y_E) has a position expressed by the equation system:

$$\begin{cases} x_E = e \\ y_E = h_2 - 2 R_2 \cos \beta \end{cases} \quad (7)$$

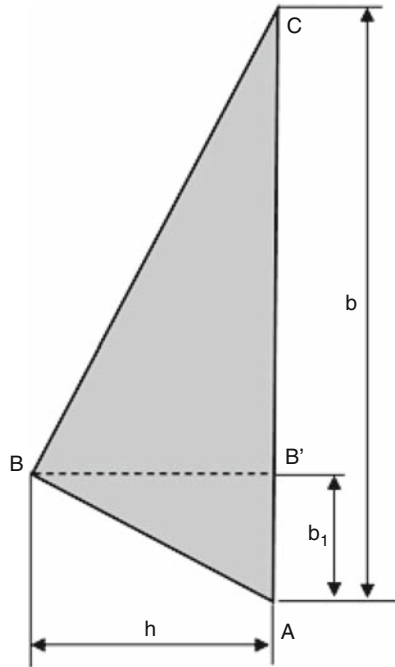


Fig. 5 Compression piston rod

In order to solve $y_E(t)$ the expression of $\beta(t)$ is needed. To obtain the function $\beta(t)$ the equation system of the BD element must be written:

$$\begin{cases} x_B + K \cos \delta + R_2 \sin \beta - e = 0 \\ y_B - K \sin \delta + R_2 \cos \beta - h_2 = 0 \end{cases} \quad (8)$$

The equation system (8) has two variables: β and δ . The variable δ is not needed, so it can be eliminated.

$$\begin{cases} x_B + R_2 \sin \beta - e = -K \cos \delta \\ y_B + R_2 \cos \beta - h_2 = K \sin \delta \end{cases} \quad (9)$$

resulting the equation:

$$\begin{aligned} x_B^2 + y_B^2 + R_2^2 + e^2 + h_2^2 + 2R_2(x_B - e) \sin \beta \\ + 2R_2(y_B - h_2) \cos \beta - 2x_B e - 2y_B h_2 = K^2 \end{aligned} \quad (10)$$

It must be applied the next substitution:

$$\begin{cases} A = 2R_2(x_B - e) \\ B = 2R_2(y_B - h_2) \\ C = K^2 + 2x_B e + 2y_B h_2 - x_B^2 - y_B^2 - R_2^2 - e^2 - h_2^2 \end{cases}$$

The equation (10) becomes simpler:

$$A \sin \beta + B \cos \beta = D \quad (11)$$

$$A \sqrt{1 - \cos^2 \beta} + B \cos \beta = D \quad (12)$$

$$1 - \cos^2 \beta = \frac{D^2}{A^2} + \frac{B^2}{A^2} \cos^2 \beta - 2 \frac{B D}{A^2} \cos \beta \quad (13)$$

$$\cos^2 \beta \left(\frac{B^2}{A^2} + 1 \right) - 2 \frac{B D}{A^2} \cos \beta + \frac{D^2}{A^2} - 1 = 0 \quad (14)$$

The second substitution will be:

$$\begin{cases} P = \frac{B^2}{A^2} + 1 \\ Q = 2 \frac{B D}{A^2} \\ R = \frac{D^2}{A^2} - 1 \end{cases}$$

resulting an algebraic equation:

$$P X^2 - Q X + R = 0 \quad (15)$$

where $X = \cos \beta$

The equation that remains, gives the β values (16) because $\beta = \beta(\theta)$ or $\beta = \beta(t)$.

$$\beta = \arccos \left[\frac{Q + \sqrt{Q^2 + 4PR}}{2P} \right] \quad (16)$$

The roots with negative sign are not practical for the mechanism; they are however in the field of *arccos* function.

Replacing β into y_E expression provides the function $y_E = y_E(t)$. There is impossible to obtain $y_E = y_E(t)$ as an analytical expression. Hence, we need to calculate y_C and y_E position point by point.

Using a computer program the moving functions $y_C = y_C(t)$ and $y_E = y_E(t)$ can be obtained. The two functions are shown in Fig. 6, y_C as y-expansion and y_E as y-compression.

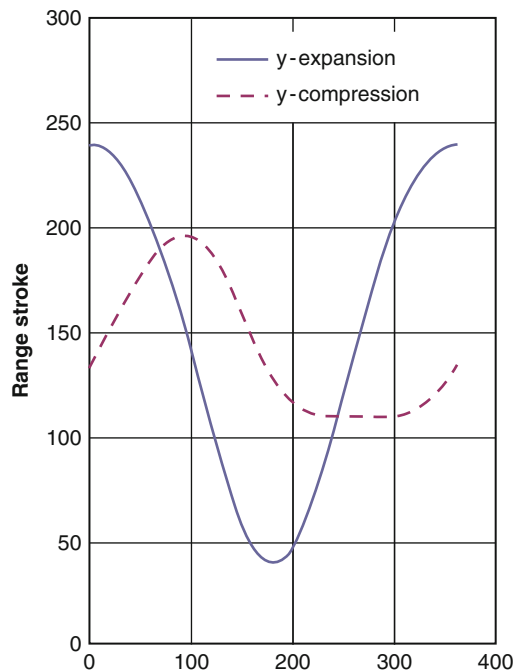


Fig. 6 Piston strokes

4 The Schmidt Analysis Insertion Paragraph

From the Schmidt analysis only the paragraphs where it is necessary to replace the old values by the new ones are shown. The original calculus presented in [5] is preserved and marked.

Therefore the mass of gas inside the regenerator space is given by:

$$(S6) \ m_r = p \cdot V_r \cdot \ln(T_h/T_c) / (R(T_h - T_c))$$

The volume of the compression space varies in sinusoidal fashion:

$$(S7) \ V_c = V_{clc} + 0.5 \cdot V_{swc} (1 + \cos(\theta))$$

θ = crank angle, proportional to time

V_{clc} = clearance volume = minimum of V_c

V_{swc} = swept volume, $\max\{V_c\} = V_{clc} + V_{swc}$

The volume of the expansion space varies in sinusoidal fashion:

$$(S8) \ V_e = V_{cle} + 0.5 \cdot V_{swe} (1 + \cos(\theta + \delta))$$

The equation (S8) has to be improved by replacing: $0.5 \cdot V_{swe} \cdot \cos(\theta + \delta)$ with $y_E(t)$.

The same operation has to be done with the equation. (S13) in Schmidt Analysis.

$$(S13) \ p = \frac{m_{tot} R T_h}{V_{ref}} \frac{1}{(e + c + d) + c \cos \theta + e \cos(\theta + \delta)}$$

Equation (S13) allows to determine the pressure p at any angle θ for given volumes and temperatures T_h and T_c .

By differentiating equation (S8) with respect to θ , the following equation from paragraph 3.1 in [5] are obtained:

$$dV_e = -0.5 \cdot V_{swe} \cdot \sin(\theta + \delta) \cdot d\theta$$

Expression “ $-0.5 \cdot V_{swe} \cdot \sin(\theta + \delta) \cdot d\theta$ ” must be replaced with a numeric differentiation of $y_E(t)$.

One has to note that the sense of the crank rotation is in accordance with the classical Schmidt analysis calculus.

5 Conclusion

The low efficiency of an α -Stirling engine drives us to improve on its work characteristics in order to enhance its efficiency. The main way to achieve this is to improve on the engine characteristics [6]. There are a lot of mechanisms species that could be used to obtain a stop movement necessary for the compression cylinder. A bar mechanism could be more reliable than a cam mechanism despite that the cam mechanism is more precise.

The bar mechanism presented in this paper has seven bars but seems to be reliable enough for this application.

There are, also, other ways to improve the efficiency, like changing the working gas in the cylinders. A good efficiency is obtained using a gas that has a high specific heat, like hydrogen or helium.

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